Changing Lanes on a Highway

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Setting: Changing lanes on a highway

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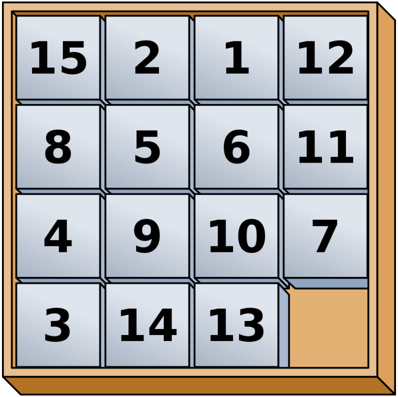
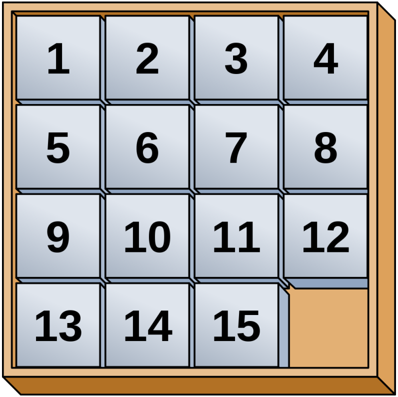
Remember the 15 puzzel?

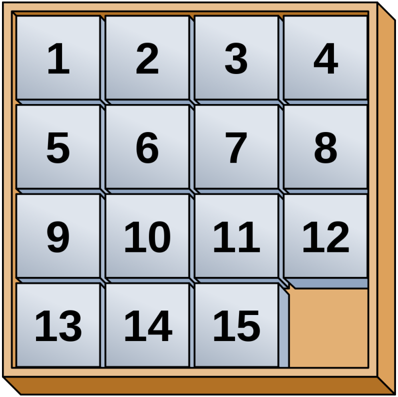


15	2	1	12
8	5	6	11
4	9	10	7
3	14	13	

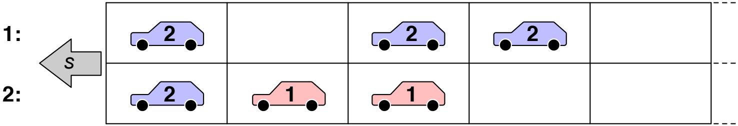
Remember the 15 puzzel?

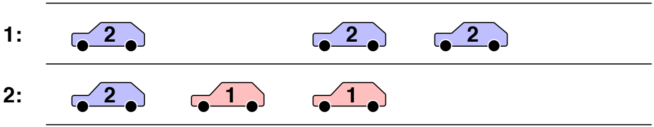


Remember the 15 puzzel?



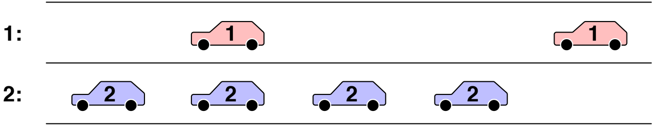
Not always solvable: Johnson and Story (1879)


1:
2:
s
2
2
2
2
1
1



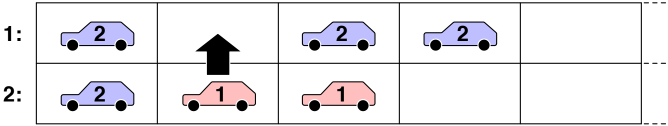
In matrix representation:

$$\begin{bmatrix} 2 & \circ & 2 & 2 & \circ & \cdots \\ 2 & 1 & 1 & \circ & \circ & \cdots \end{bmatrix}$$



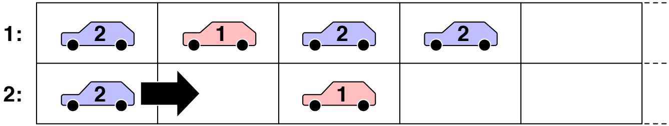
In matrix representation:

$$\begin{bmatrix} \circ & 1 & \circ & \circ & 1 & \cdots \\ 2 & 2 & 2 & 2 & \circ & \cdots \end{bmatrix}$$



Legal move: *switch*

$$\begin{bmatrix} 2 \\ \circ \end{bmatrix} \mapsto \begin{bmatrix} \circ \\ 2 \end{bmatrix}, \quad \begin{bmatrix} \circ \\ 1 \end{bmatrix} \mapsto \begin{bmatrix} 1 \\ \circ \end{bmatrix}.$$

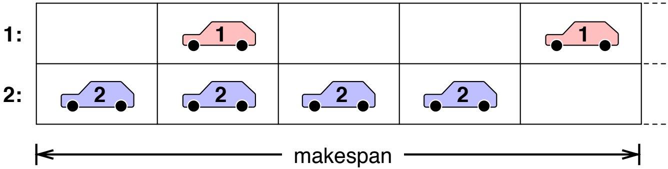


Legal move: *delay*

$$\begin{bmatrix}1 & \circ\end{bmatrix} \mapsto \begin{bmatrix}\circ & 1\end{bmatrix}, \quad \begin{bmatrix}2 & \circ\end{bmatrix} \mapsto \begin{bmatrix}\circ & 2\end{bmatrix}.$$

Objective Functions

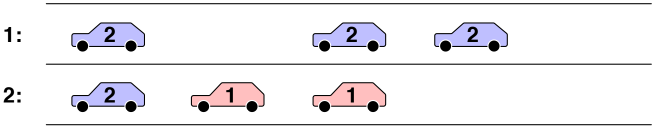
A solution is feasible if all cars are on their desired lane:



Objective Functions:

- Minimise the number of moves.
- Minimise the makespan.

Notation



$\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ is called a $(2,1)$-pair, or tricky pair.

Problems

We also consider two additional variants of the original problem.

P0. The original lane-changing problem, as defined before.

P1. A restricted version of P0: a solution is feasible only if each car that was involved in a $(2, 1)$-input pair has been delayed at least once.

P2. A relaxed version of P1: multiple cars may occupy the same slot in intermediate configurations.

What is the tricky part?

Tricky:

$$\begin{bmatrix} 2 & \circ \\ 1 & \circ \end{bmatrix} \mapsto \begin{bmatrix} 2 & \circ \\ \circ & 1 \end{bmatrix} \mapsto \begin{bmatrix} \circ & \circ \\ 2 & 1 \end{bmatrix} \mapsto \begin{bmatrix} \circ & 1 \\ 2 & \circ \end{bmatrix}.$$

Easy:

$$\begin{bmatrix} 2 & \circ \\ 2 & \circ \end{bmatrix} \mapsto \begin{bmatrix} 2 & \circ \\ \circ & 2 \end{bmatrix} \mapsto \begin{bmatrix} \circ & \circ \\ 2 & 2 \end{bmatrix}.$$

Solving P2 and P1

$$\begin{bmatrix} 2 & \circ & 2 & 2 & \circ & \cdots \\ 2 & 1 & 1 & \circ & \circ & \cdots \end{bmatrix}$$

Algorithm A2 for solving P2:

1. Iterate over the columns and move the cars along the flow.

Algorithm A1 for solving P1:

1. Compute flow using A2.
2. Pull cars from the back according to the flow.

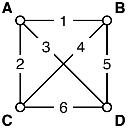
Solving P0

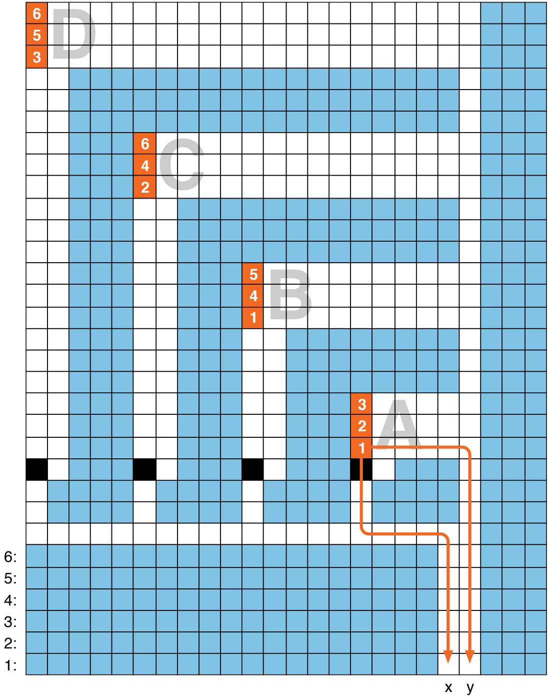
Corollary 6. Let Y be a solution for P1 that is simultaneously makespan-optimal and cost-optimal. Then Y is also a feasible makespan-optimal solution for P0. Furthermore, the total number of moves in Y is at most 1.5 times the total number of moves in a cost-optimal P0 solution.

NP-hardness

Now we increase the number of lanes.

- Reduction from Minimum Vertex Cover on 3-regular graphs.
- Use agents to block the way as *'switch'*.
- An optimum solution needs to find the optimum number of switches, which is in our construction equivalent to finding a Minimum Vertex Cover.


A
B
1
3
4
2
5
C
6
D


6
5
3
D
6
4
2
C
5
4
1
B
3
2
1
A
6:
5:
4:
3:
2:
1:
x
y

Questions?

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- Jukka Suomela